3. NUMERIC EXAMPLE

This document presents a complete numerical example for the document "*Proposal for the risk model (using standard modelling and equations)*". This example will be based on one of the settling problems evaluated by the risk model, i.e. the filamentous bulking caused by low DO concentrations. Three variables are involved in this case: Food to microorgansims ratio (FtoM), dissolved oxygen (DO) and the risk of bulking.

3.1 Fuzzyfication

First of all, the information involving the membership functions (MF) is presented. Figures 10 and 11 illustrate the graphical representation of the MFs.

FtoM MF

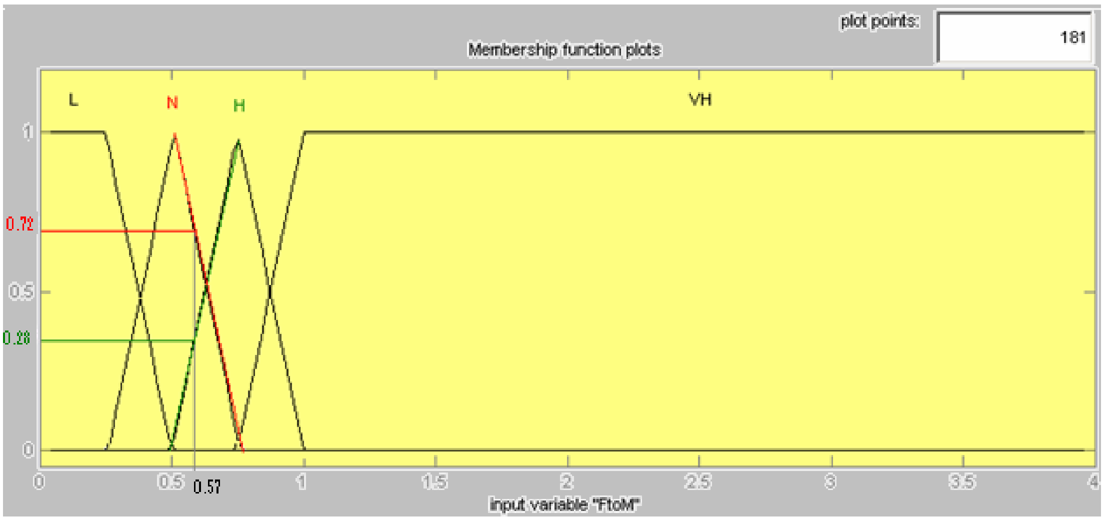

Figure 10. MFs for the input "food to microorganisms ratio". L=Low, N= Normal, H= High and VH=Very High. In green: membership to the MF 'High'; In red: membership to the MF 'Normal'.

These MFs (Figure 10) are converted to equations as follows:

$$MF_{Low}^{F\,to\,M}(FtoM,-0.25,0,0.25,0.5) = \max\left(\min\left(\frac{FtoM-(-0.25)}{0-(-0.25)},1,\frac{0.5-FtoM}{0.5-0.25}\right),0\right)$$

$$MF_{Normal}^{F\,to\,M}(FtoM,0.25,0.5,0.75) = \max\left(\min\left(\frac{FtoM-0.25}{0.5-0.25},\frac{0.75-FtoM}{0.75-0.5}\right),0\right)$$

$$MF_{High}^{F\,to\,M}(FtoM,0.5,0.75,1) = \max\left(\min\left(\frac{FtoM-0.5}{0.75-0.5},\frac{1-FtoM}{1-0.75}\right),0\right)$$

$$MF_{VeryHigh}^{F\,to\,M}(FtoM,0.75,1,4,4.5) = \max\left(\min\left(\frac{FtoM-0.75}{1-0.75},1,\frac{4.5-FtoM}{4.5-4}\right),0\right)$$

DO MF

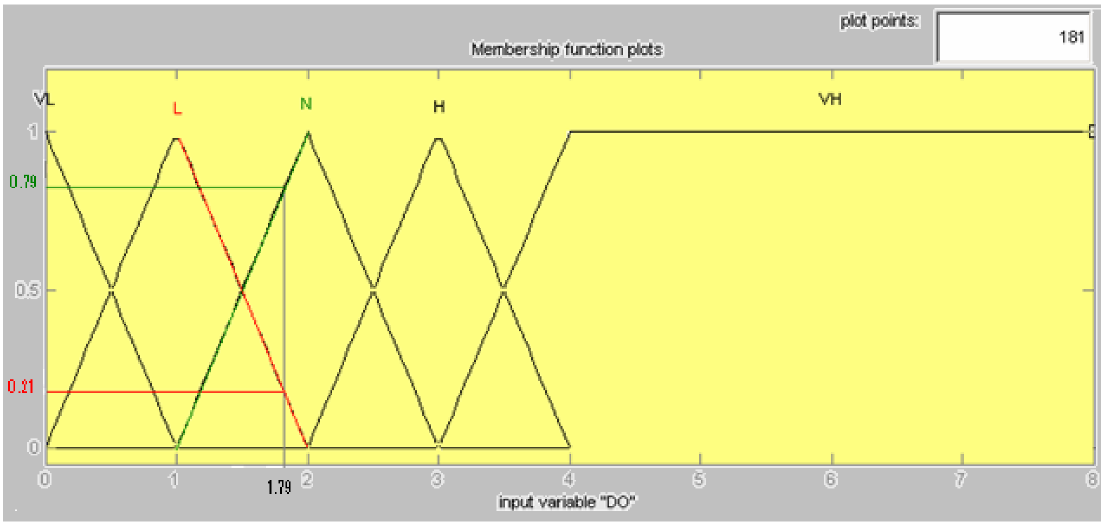

Figure 11. MFs for the input “DO”. VL= Very Low, L=Low, N= Normal, H= High and VH=Very High. In green: membership to the MF ’Normal’; In red: membership to the MF ’Low’.

These are the equations of the MF for input ‘DO’, presented in Figure 11:

$$MF_{Very\ Low}^{DO}(DO,-0.5,-0.25,0,1) = \max\left(\min\left(\frac{DO-(-0.5)}{(-0.25)-(-0.5)},1,\frac{1-DO}{1-0}\right),0\right)$$

$$MF_{Low}^{DO}(DO,0,1,2) = \max\left(\min\left(\frac{DO-0}{1-0},\frac{2-DO}{2-1}\right),0\right)$$

$$MF_{Normal}^{DO}(DO,1,2,3) = \max\left(\min\left(\frac{DO-1}{2-1},\frac{3-DO}{3-2}\right),0\right)$$

$$MF_{High}^{DO}(DO,2,3,4) = \max\left(\min\left(\frac{DO-2}{3-2},\frac{4-DO}{4-3}\right),0\right)$$

$$MF_{Very\ High}^{DO}(DO,3,4,8,8.5) = \max\left(\min\left(\frac{DO-3}{4-3},1,\frac{8.5-DO}{8.5-8}\right),0\right)$$

Step 1:

Now, as an example, the fuzzification will be explained with real values for the two input variables. Imagine that for a given time instant, we have the following values for the two input variables (‘FtoM’ and ‘DO’):

FtoM = 0.57 Kg $COD_{removed}$/Kg MLVSS·d

DO = 1.79 mg.O_2 / L.

The first step is to determine the degree of membership to each MF for each input variable.

Taking the equations for the FtoM input we can calculate,

for the ‘Low’ MF:

$$MF_{Low}^{F\,to\,M}\left(FtoM,-0.25,0,0.25,0.5\right)=\max\left(\min\left(\frac{FtoM-(-0.25)}{0-(-0.25)},1,\frac{0.5-FtoM}{0.5-0.25}\right),0\right)$$

$$for\ FtoM=0.57$$

$$MF_{Low}^{F\,to\,M}\left(0.57,-0.25,0,0.25,0.5\right)=y_{Low}^{F\,to\,M}=\deg ree\,of\ membership=0$$

for the ‘normal’ MF:

$$MF_{Normal}^{F\,to\,M}\left(FtoM,0.25,0.5,0.75\right)=\max\left(\min\left(\frac{FtoM-0.25}{0.5-0.25},\frac{0.75-FtoM}{0.75-0.5}\right),0\right)$$

$$forFtoM=0.57$$

$$MF_{Normal}^{F\,to\,M}\left(0.57,0.25,0.5,0.75\right)=y_{Normal}^{F\,to\,M}=\deg ree\,of\ membership=0.72$$

for the ‘high’ MF:

$$MF_{High}^{F\,to\,M}\left(FtoM,0.5,0.75,1\right)=\max\left(\min\left(\frac{FtoM-0.5}{0.75-0.5},\frac{1-FtoM}{1-0.75}\right),0\right)$$

$$for\ x=0.57$$

$$MF_{High}^{F\,to\,M}\left(0.57,0.5,0.75,1\right)=y_{High}^{F\,to\,M}=\deg ree\,of\ membership=0.28$$

and for the ‘Very high’ MF:

$$MF_{VeryHigh}^{F\,to\,M}\left(FtoM,0.75,1,4,4.5\right)=\max\left(\min\left(\frac{FtoM-0.75}{1-0.75},1,\frac{4.5-FtoM}{4.5-4}\right),0\right)$$

$$for\ x=0.57$$

$$MF_{VeryHigh}^{F\,to\,M}\left(FtoM,0.75,1,4,4.5\right)=y_{VeryHigh}^{F\,to\,M}=\deg ree\,of\ membership=0$$

Taking the equations for the DO input we can calculate,
for the ‘Very low’ MF:

$$MF_{Very\,Low}^{DO}\left(DO,-0.5,-0.25,0,1\right)=\max\left(\min\left(\frac{DO-(-0.5)}{(-0.25)-(-0.5)},1,\frac{1-DO}{1-0}\right),0\right)$$

$$for\ x=1.79$$

$$MF_{Very\,Low}^{DO}\left(1.79,-0.5,-0.25,0,1\right)=y_{Verylow}^{DO}=\deg ree\,of\ membership=0$$

for the ‘low’ MF:

$$MF_{Low}^{DO}\left(DO,0,1,2\right)=\max\left(\min\left(\frac{DO-0}{1-0},\frac{2-DO}{2-1}\right),0\right)$$

$$for\ x=1.79$$

$$MF_{Low}^{DO}\left(1.79,0,1,2\right)=y_{low}^{DO}=\deg ree\,of\ membership=0.21$$

for the ‘normal’ MF:

$$MF^{DO}_{Normal}(DO,1,2,3) = \max\left(\min\left(\frac{DO-1}{2-1},\frac{3-DO}{3-2}\right),0\right)$$

$$for\ x = 1.79$$

$$MF^{DO}_{Normal}(1.79,1,2,3) = y^{DO}_{Nrormal} = \deg ree\, of\ membership = 0.79$$

for the 'high' MF:

$$MF^{DO}_{High}(DO,2,3,4) = \max\left(\min\left(\frac{DO-2}{3-2},\frac{4-DO}{4-3}\right),0\right)$$

$$for\ x = 1.79$$

$$MF^{DO}_{High}(1.79,2,3,4) = y^{DO}_{High} = \deg ree\, of\ membership = 0$$

and for the 'Very high' MF:

$$MF^{DO}_{Very\ High}(DO,3,4,8,8.5) = \max\left(\min\left(\frac{DO-3}{4-3},1,\frac{8.5-DO}{8.5-8}\right),0\right)$$

$$for\ x = 1.79$$

$$MF^{DO}_{Very\ High}(1.79,3,4,8,8.5) = y^{DO}_{Veryhigh} = \deg ree\, of\ membership = 0$$

To sum up, we can see that for the FtoM value we have a certain degree of membership to the 'normal' MF (0.72) and to the 'high' MF (0.28). For the DO we can see that we have a certain degree of membership to the 'low' MF (0.21) and to the 'normal' MF (0.79), i.e.

$$F\ to\ M\begin{cases} Degree\, of\ membership\, of\ 0.72\, to\, the\, 'normal'\, MF \\ Degree\, of\ membership\, of\ 0.28\, to\, the\, 'high'\, MF \end{cases}$$

$$DO\quad\begin{cases} Degree\, of\ membership\, of\ 0.21\ to\, the\, 'low'\, MF \\ Degree\, of\ membership\, of\ 0.79\ to\, the\, 'normal'\, MF \end{cases}$$

Figures 10 and 11 illustrate graphically the membership degrees for FtoM and DO, respectively.

3.2 Rule base (decision matrix) definition

From the decision matrix (see Table 2, below) of the risk model knowledge base (pages 19 to 22), the rules are obtained and represented as follows:

Table 2. Representation of a decision matrix.

		DO				
		Very Low	**Low**	**Normal**	**High**	**Very High**
F to M	**Low**	Low	Low	Low	Low	Low
	Normal	High	Normal	Low	Low	Low
	High	High	High	Normal	Low	Low
	Very High	High	High	High	Normal	Low

For example: From the decision matrix of Table 2 we can obtain the following rules:

IF *F to M* is **Low** and *DO* is **Normal**, THEN *Risk of Low DO bulking* is **Low**.

Step 2:

We have already seen that the input variables have the following memberships:

FtoM belongs to both ‘normal’ and ’high’ MF (with different degrees to each one).
DO belongs to both ‘low’ and ‘normal’ MF (also with different degrees to each one).

Now, according to the decision matrix, it is necessary to know which rules will be satisfied. With FtoM normal and high (in different degrees) and DO low and normal (in different degrees), the rules satisfied are (for more details see Figure 12 below):

1. IF *FtoM* is **Normal** and *DO* is **Low**, THEN *Risk of Low DO bulking* is **Normal (Rule 12)**
2. IF F*toM* is **Normal** and *DO* is **Normal**, THEN *Risk of Low DO bulking* is **Low (Rule 13)**
3. IF *FtoM* is **High** and *DO* is **Low**, THEN *Risk of Low DO bulking* is **High (Rule 7)**
4. IF *FtoM* is **High** and *DO* is **Normal**, THEN *Risk of Low DO bulking* is **Normal (Rule 8)**

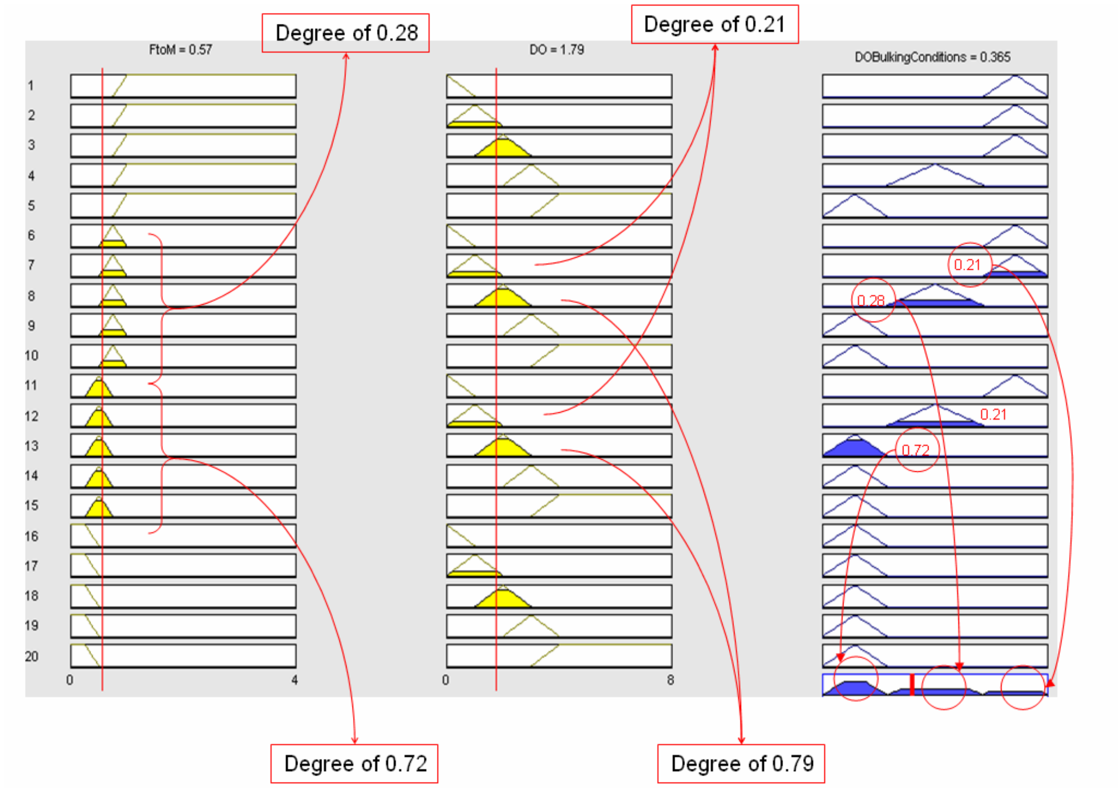

Figure 12. Example of the satisfied rules and chosen degrees of membership for fuzzify and defuzzyfy

3.3 Defuzzyfication

As said above (see **2.3 Defuzzyfication**), for the Risk model the shape and ranks of the MFs for the output are always the same. Therefore, the MFs for the risk of low DO bulking (as well as for the other risks of problems) can be defined with the following functions:

$$MF_{Low}^{Risk\ of\ problem}\begin{cases} y = 5x_1 + 1 \\ y = -5x_2 + 1 \end{cases}$$

$$MF_{Normal}^{DO}\begin{cases} y = \frac{1}{0.3}x_1 - 0.665 \\ y = -\frac{1}{0.3}x_2 + 2.665 \end{cases}$$

$$MF_{High}^{Risk\ of\ \ problem}\begin{cases} y = 5x_1 - 4 \\ y = -5x_2 + 6 \end{cases}$$

These functions will allow calculating the risk of filamentous Bulking due to low DO.

Step 3:

We have to look rule by rule, each one of the four satisfied rules (step 2), i.e. to determine the degree of membership of the output for the first satisfied rule:

IF *FtoM* is **Normal** and *DO* is **Low**, THEN *Risk of* l*ow DO bulking* is **Normal**

We have to take the lower degree of membership among the degrees of membership of the inputs, i.e. since the FtoM has a degree of membership of 0.72 to the MF ‘Normal’ (remember **step 1**, above) and the DO has a degree of membership of 0.21 to the MF ‘Low’, then, for this first rule, the Risk of low DO bulking will have a degree of membership equal to 0.21 for the Normal MF.

The same must be done for the other 3 rules satisfied of step 2 and the results can be summarized as:

Rule 12: Risk of bulking: MF ‘Normal’ (0.21)

Rule 13: Risk of bulking: MF ‘Low’ (0.72)

Rule 7: Risk of bulking: MF ‘High’ (0.21)

Rule 8: Risk of bulking: MF ‘Normal’ (0.28)

For each Risk of bulking affecting to the same MF of the output (i.e., Rules 12 and 8 affecting to the MF ‘Normal’), we will choose the one with the highest degree of membership. So, we will take:

Rule 13: Risk of bulking: MF ‘Low’ (0.72)

Rule 7: Risk of bulking: MF ‘High’ (0.21)

Rule 8: Risk of bulking: MF ‘Normal’ (0.28)

Now we can start calculating the areas of the figures formed in Figure 12 and calculate the centroid of the “integrated” figure:

1. The area of the figure resulting in the MF ‘Low’ (with a degree of membership of 0.72) of the output (Risk of Bulking) can be calculated with the formula (the degrees of membership to each MF of the output is equal to the **y** of the functions for the output variable):

$$MF_{Low}^{Risk\ of\ bulking}\begin{cases} y = 5 \cdot x_1 + 1 \\ y = -5 \cdot x_2 + 1 \end{cases}$$

$$y = 0.72 = \begin{cases} x_1 = -0.056 \\ x_2 = 0.056 \end{cases}$$

$$A_{Low} = \frac{[0.4 + (x_2 - x_1)] \cdot y_{Low}}{2}$$

$$A_{Low} = \frac{[0.4 + (0.056 - (-0.056))] \cdot 0.72}{2}$$

$$A_{Low} = 0.18432$$

The same procedure is carried out for the Rule 7 (giving a Risk of bulking with a degree of membership to the MF ‘High’ of 0.21) and Rule 8 (giving a Risk of bulking with a degree of membership to the MF ‘Normal’ of 0.28).

2. MF ‘High’ (0.21)

$$MF_{High}^{Risk\ of\ bulking} \begin{cases} y = 5 \cdot x_1 - 4 \\ y = -5 \cdot x_2 + 6 \end{cases}$$

$$y = 0.21 = \begin{cases} x_1 = 0.842 \\ x_2 = 1.158 \end{cases}$$

$$A_{High} = \frac{[0.4 + (x_2 - x_1)] \cdot y_{High}}{2}$$

$$A_{High} = \frac{[0.4 + (1.158 - 0.842)] \cdot 0.21}{2}$$

$$A_{High} = 0.07518$$

3. MF ‘Normal’ (0.28):

$$MF_{Normal}^{Risk\ of\ bulking} \begin{cases} y = \dfrac{1}{0.3} x_1 - 0.665 \\ y = -\dfrac{1}{0.3} x_2 + 2.665 \end{cases}$$

$$y = 0.28 = \begin{cases} x_1 = 0.2835 \\ x_2 = 0.7155 \end{cases}$$

$$A_{Normal} = \frac{[0.6 + (x_2 - x_1)] \cdot y_{Normal}}{2}$$

$$A_{Normal} = \frac{[0.6 + (0.7155 - 0.2835)] \cdot 0.28}{2}$$

$$A_{Normal} = 0.14$$

Finally, applying the formula of the centroid we can calculate the final percentage for the risk of bulking due to Low DO concentration:

$$x_c = \frac{A_{Low} \cdot 0 + A_{Normal} \cdot 0.5 + A_{High} \cdot 1}{A_{Low} + A_{Normal} + A_{High}}$$

$$x_c = \frac{0.18 \cdot 0 + 0.14 \cdot 0.5 + 0.075 \cdot 1}{0.18 + 0.14 + 0.075}$$

$$x_c = 0.36 \cdot 100 = 36\%\ Risk\ of\ Bulking$$